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Waves in rods

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An essential introduction for students and researchers, particularly helpful because of the exhaustive bibliography--EDITOR

Despite some theoretical interest in the problem of waves in confined media (such as circular rods) during the 19th century [1-3], little experimental work was performed on this topic until the early 20th century [4, 5]. This pioneering work was concerned with the determination of the shapes of pressure pulses produced by the impact of bullets or the explosion of detonators and was quickly put to use during the First World War [6]. The device consisting of a long rod used to transmit a force pulse to somewhere where it can be measured became known as the Hopkinson pressure bar after its inventor. It is still sometimes used for its original purpose of studying pressure pulses in explosions [7-13].

Interest in the (complicated) problem of the transmission of waves down rods revived during the Second World War due to the use of rods as mechanical waveguides and delay lines in a variety of applications including early radar machines [14-24]. As microphone technology had been used to interconvert electromagnetic and mechanical waves, the first Hopkinson bar devices built after the War to test the mechanical properties of materials at high strain rates used capacitance microphones to detect the strain pulses in the bars [25, 26].

Commercial strain gauges had been available since the 1930s [27-29] and were used during the Second World War to study various dynamic problems including the impulses delivered to aircraft landing gear on landing [30, 31] and high strain rate tensile testing [32, 33]. However, there was initially concern that a gauge bonded to the outside of a bar would not give a faithful record of a strain pulse, particularly if this pulse were launched by a non-uniform load distribution at the bar end [34, 35].

This issue was resolved by theoretical work and experimental checks on whether the Saint Venant Principle could be extended to dynamic 'non-equilibrium' loading problems [36-41] And from about 1953 onwards it became standard to use strain gauges bonded to the outside of Hopkinson bars to measure strain pulses propagating down them [42-46]. All of this opened the way to the compression Hopkinson bar becoming a more widely used and accepted tool in the dynamic testing of materials [47, 48].

Most workers in the field of split Hopkinson bar testing assume that a one-dimensional wave analysis of the system is good enough [49-51]. The mathematics of the full three-dimensional analysis is very complex and using it would probably yield an increase in accuracy too small to be worth the effort. The main results of the analysis is the explanation of the 'ringing' seen on signals after they have propagated down the rods. This is a due to signals of different frequency travelling at different speeds (dispersion). Over the years a number of researchers have worked at the problem, trying to create a dispersion correction algorithm simple enough for routine use. Notable contributions to this area after the seminal work of [1, 2, 15, 19] include: [52-68]. If rods had continued to be used as mechanical waveguides, an algorithm would probably have been developed by now.

The effect of temperature gradients on the propagation of waves down rods has recently become important due to the desire to check constitutive models of materials over a wide range of temperatures and strain rates. There are three main effects due to the change in elasticity of the material of the bar with temperature: (i) the particle velocity at the end of a bar for a given force will change [69, 70]; (ii) the impedance change due to the temperature gradient will result in some of the elastic wave energy being reflected [71, 72]; and (iii) elastic waves propagating through the temperature gradient will be distorted differently by dispersion compared to a rod all at the same temperature [73-79]. Fortunately the errors introduced are small so long as the test temperature is no more than about 300 °C removed from room temperature [80-84]. Thus low temperature tests, even down to liquid helium temperatures [85], can be performed with bars made from standard materials such as steel.

Various approaches have been tried: (i) tapering the bar to compensate for the temperature induced impedance change [72, 86, 87] (this method has the disadvantage that a bar of a particular profile can only compensate for one particular temperature gradient); (ii) calculating the effect of the temperature gradient on the wave propagation [46, 69]; (iii) measuring the effect of the temperature gradient on the wave propagation [70, 71]. All three methods assume the temperature gradient is in a steady state. If it is desired to heat a specimen rapidly by induction heating, for example, [88], none of these methods can be used.

If it is desired to test materials above +300 °C, the bars should be made from a material whose elastic properties are only a weak function of temperature e.g. Inconel 718 [89]. However, the maximum temperature this material can be used at before its elastic properties start to decline sharply is +600°C. Above this temperature, the specimen must be kept thermally insulated from the bars. Two ways of doing this have been devised. The first is to place thermal insulation between the specimen and the bars [90, 91]. The second is to construct a mechanical device that brings the (cold) bars into contact with the heated specimen a fraction of a second before the stress pulse arrives at the end of the input rod [92]. This last approach is probably the only practical method of testing materials at high strain rates above 1000 °C.

The desire to obtain the high strain-rate mechanical properties of weak materials such as polymer foams and polymer bonded explosives has led several groups to develop low impedance polymer Hopkinson bar systems [93-101]. The analysis of wave distortion due to dispersion is more complex than for metal bars, even in the absence of temperature gradients.
So far it has been implicitly assumed that the rods have been circular in cross-section. Only one experimenter to my knowledge has used non-circular bars in Hopkinson bar testing [102]. However, the problem of waves propagating down rectangular or elliptical rods is of wide importance in structural and earthquake engineering. A few references are given as a way into this literature: [20, 103-129].

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